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Nonsimilar Free Convection Boundary Layer in Non-Newtonian Fluid-Saturated Porous Media

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Finite difference method with and without the local similarity assumption are described for the nonsimilar free convection boundary-layer flow over a body of arbitrary shape, embedded in a non-Newtonian fluid-saturated porous medium, with an arbitrarily varying surface temperature. For illustration, numerical results are obtained with and without the local similarity assumption for the case of a vertical flat plate with exponential, power-law, and sinusoidal surface temperature distributions. A simple integral method is also proposed, and its results are compared against those from the exact as well as the finite difference solutions. Comparison of the results reveals that all proposed integral and numerical methods perform equally well for both dilatant and pseudoplastic fluids.

Nomenclature

f = dimensionless stream function
g = acceleration due to gravity
g_x = component of acceleration due to gravity in the x direction
I = function defined in Eq. (12)
K = intrinsic permeability of the porous media
m_t = parameter associated with wall temperature, defined in Eq. (19)
Nu_x = local Nusselt number
Ra_x = local Rayleigh number, defined in Eq. (13)
r = function representing wall geometry
r^* = 1 for plane flow and r for axisymmetric flow
T = temperature
T_e = ambient constant temperature
T_w = variable wall surface temperature
u, v = Darcian or superficial velocity components
x, y = boundary-layer conditions
α = equivalent thermal diffusivity of the fluid-saturated porous media
β = expansion coefficient of fluid
ΔT = temperature difference, $T_w - T_e$
δT = thermal boundary-layer thickness
ζ = variable defined in Eq. (29)
η = similarity variable defined in Eq. (12)
θ = dimensionless temperature difference
λ = exponent introduced in Eq. (21)
μ = fluid viscosity
ν = fluid kinematic viscosity
ρ = fluid density
ψ = stream function

Subscripts
e = ambient
w = wall

Introduction

SINCE the celebrated analysis of Cheng and Minkowycz[1] for free convection over a vertical semi-infinite flat plate in a Newtonian fluid-saturated porous medium, a considerable number of theoretical and experimental investigations have been carried out to explore the complex phenomena associated with convection in porous medium. Excellent reviews may be found in Cheng[2] and Nield and Bejan.[3]

There is no denying that a number of industrially important fluids such as fossile fuels in saturated underground beds exhibit non-Newtonian fluid behavior. Despite its importance in practical engineering application, only a limited number of studies have been reported on free convection in non-Newtonian fluid-saturated porous media. Chen and Chen[4,5] solved the boundary-layer equations for free convection of non-Newtonian fluids over a vertical flat plate, a horizontal circular cylinder, and a sphere in porous media, using the power-law model proposed by Christopher and Middleman,[6] while a general similarity transformation procedure has been proposed by Nakayama and Koyama[7] to find a class of possible similarity solutions for free convection flow of non-Newtonian fluids over a nonisothermal body of arbitrary shape in porous media. A unified treatment for non-Darcy mixed convection in power-law fluid-saturated porous media has been recently proposed by Nakayama and Shenoy.[8] All these studies, however, are limited to the cases where similarity solutions are possible. In many practical situations, similarity solutions are not permitted, and therefore one has to resort to a more general treatment, such as a local similarity or local nonsimilarity solution methodology as proposed by Sparrow and Yu[9] for the classical thermal boundary-layer problems of clear fluid (i.e., without porous media). Alternatively, one may appeal to a simple and yet fairly accurate integral method, proposed by Nakayama and Koyama[7] for the analysis of non-Newtonian free convection from a body of arbitrary shape.

In this study, two distinct finite difference methods based on the local similarity and local nonsimilarity solutions are proposed for analyzing nonsimilar free convection boundary-layer flow over a body of arbitrary shape embedded in a non-Newtonian fluid-saturated porous medium, with an arbitrary varying surface temperature. Several computational examples are provided and the results based on the two finite difference methods and the approximate integral method are examined to assess their performance in the nonsimilar thermal boundary-layer analysis for free convection in non-Newtonian fluid-saturated porous media.

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Physical Model and Governing Equations

Consider Darcy free convective flow about a plane or axisymmetric smooth impermeable body with arbitrary surface temperature $T_w(x)$ embedded in a porous medium saturated with a non-Newtonian fluid, as shown in Fig. 1. Assuming that the Rayleigh number is sufficiently high for the boundary-layer approximations to be valid, the governing equations, namely, the continuity equation, the Darcy's law with the Boussinesq approximation, and the energy equation, are given by

$$\frac{\partial r^* u}{\partial x} + \frac{\partial r^* v}{\partial y} = 0 \tag{1}$$

where

$$r^* = \begin{cases} 1: & \text{plane flow} \\ r(x): & \text{axisymmetric flow} \end{cases} \tag{2}$$

The non-Newtonian power laws proposed by Christopher and Middleman[6] and Dharmandhikari and Kale[10] have a common form, namely

$$-\frac{dp}{dx} = \frac{\mu}{K(n)} u^n + \rho g_x \tag{3}$$

where μ is the power law constant, while $K(n)$ as a function of the power law index n is a modified permeability, which is given for the case of packed spheres as

$$K(n) = \begin{cases} \dfrac{6}{25}\left(\dfrac{n\varepsilon}{3n+1}\right)^n \left(\dfrac{\varepsilon d}{3(1-\varepsilon)}\right)^{n+1} & \text{(Christopher and Middleman)} \\ \dfrac{2}{\varepsilon}\left(\dfrac{d\varepsilon^2}{8(1-\varepsilon)}\right)^{n+1} \dfrac{6n+1}{10n-3}\left(\dfrac{16}{75}\right)^{[3(10n-3)/(10n+11)]} & \text{(Dharmandhikari and Kale)} \end{cases} \tag{4}$$

where d is the particle diameter, while ε is the porosity. A general expression valid for consolidated porous media can be found in Wang and Tu,[11] who also presented rigorous derivation of Eq. (3) starting from a fundamental expression for the apparent viscosity, with the power-law exponent, consistency index, and yield stress as rheological parameters.

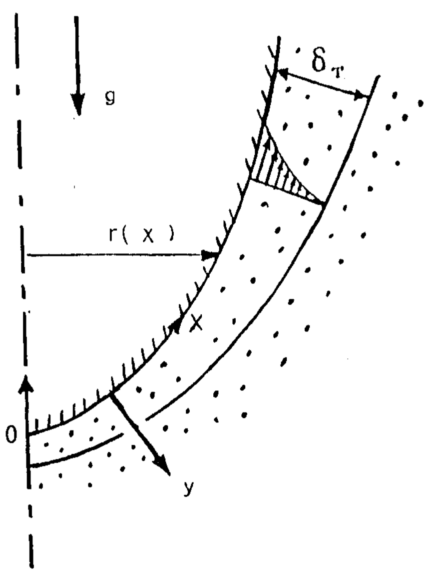

Fig. 1 Coordinate system.

g_x is related to the body shape function $r(x)$ is given by

$$g_x = g\left[1 - \left(\frac{dr}{dx}\right)^2\right]^{1/2} \tag{5}$$

We assume local thermodynamic equilibrium between the fluid and solid phases, and consider the energy equation as follows:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{6}$$

where α is the effective thermal diffusivity of the fluid-solid system. The boundary conditions are

$$y = 0: \quad v = 0; \quad T = T_w(x) \tag{7}$$

$$y = \infty: \quad u = 0; \quad T = T_e \tag{8}$$

where, as usual, the subscripts w and e refer to the wall and boundary-layer edge, respectively.

Let us introduce the Boussinesq approximation to Eq. (3), and eliminate the pressure gradient term using the above boundary conditions. Thus, we may replace Eq. (3) by

$$u^n = \rho K g_x \beta(T - T_e)/\mu \tag{9}$$

Local Similarity and Nonsimilarity Equations

Nakayama and Koyama[7] introduced the following transformations:

$$\psi = \alpha r^* (Ra_x I)^{1/2} f(x, \eta) \tag{10}$$

$$T - T_e = \Delta T \theta(x, \eta) \tag{11}$$

$$\eta = (y/x)(Ra_x/I)^{1/2} \tag{12}$$

where

$$I(x) = \frac{\int_0^x \Delta T^{(2n+1)/n} g_x^{1/n} r^{*2}\, dx}{\Delta T^{(2n+1)/n} g_x^{1/n} r^{*2} x} \tag{13}$$

$$Ra_x = \frac{x}{\alpha}\left(\frac{K\beta\Delta T g_x}{\nu}\right)^{1/n} \tag{14}$$

is the local Rayleigh number based on the local values of g_x and

$$\Delta T(x) = T_w(x) - T_e \tag{15}$$

η is the proposed pseudosimilarity variable which contains the function I as defined by Eq. (13) to adjust the scale in the normal direction according to a given geometry and its surface temperature distribution. Obviously, the function I

reduces to unity for the special case of an isothermal flat plate. Substitutions of Eqs. (10–12) into Eqs. (6–9), yields

$$f'^n = \theta \tag{16}$$

$$\theta'' + \left(\frac{1}{2} - m_t I\right) f\theta' - m_t I f'\theta = xI\left[f'\frac{\partial\theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right] \tag{17}$$

with the boundary conditions transformed as

$$\eta = 0: \quad f = 0, \quad \theta = 1 \tag{18}$$

$$\eta = \infty: \quad \theta = 0 \tag{19}$$

where

$$m_t(x) = \frac{\mathrm{d}\,\ell n\,\Delta T}{\mathrm{d}\,\ell n\,x} \tag{20}$$

The primes in the above equations denote differentiations with respect to η.

For the general isothermal case in which the wall temperature does not follow the relation given by

$$\Delta T \propto \left(\int_0^x g_x^{1/n} r^{*2}\,\mathrm{d}x\right)^\lambda \tag{21}$$

the product $m_t I$ in Eq. (17) varies in the streamwise direction. Thus, similarity solutions are no longer possible. To treat such a general situation, either the local nonsimilarity method, the local similarity method of Sparrow and Yu,[9] or the finite difference method may be exploited. The local similarity assumption holds for most of boundary-layer equations, provided that the equations are scaled (or transformed) properly.

Under the local similarity assumption, the terms on the right side of Eq. (17) may be dropped to get

$$\theta'' + (\tfrac{1}{2} - m_t I) f\theta' - m_t I f'\theta = 0 \tag{22}$$

Equation (13) reveals that the energy Eq. (22) reduces to a similarity equation, when $\Delta T \propto \xi^\lambda$. Substitution of this relation into Eq. (13), and then to the energy Eq. (22), yields the following similarity equation for the energy distribution:

$$\theta'' + \frac{n+\lambda}{2[n+(2n+1)\lambda]} f\theta' - \frac{n\lambda}{n+(2n+1)\lambda} f'\theta = 0 \tag{23}$$

The resulting equation, together with the Eq. (16) and the boundary conditions [Eq. (18)], is identical to one derived by Nakayama and Koyama[7] for a vertical plate with its wall temperature varying as $\Delta T \propto \xi^\lambda$, which has been investigated employing the Runge-Kutta-Gill integration scheme as well as an approximate integral method. In the present analysis, the local similarity Eqs. (16) and (22) satisfying the boundary conditions [Eq. (18)] are investigated in the following sections, employing the Runge-Kutta-Gill integration scheme together with the Nachtsheim-Swigert iteration scheme of Nachtsheim and Swigert,[12] as well as the implicit finite difference scheme along with the Keller box method of Keller.[13] In addition to these, solutions of the energy Eq. (6) satisfying the boundary conditions [Eqs. (7) and (8)] are also proposed to be obtained by the approximate integral method of Nakayama and Koyama.[14] Finally, the local nonsimilarity equation for the energy distribution [Eq. (17)] will also be obtained by the two-point finite difference method.

The following formula can be used for the general nonisothermal case:

$$Nu_x = -\theta'(0; m_t I)(Ra_x/I)^{1/2} \tag{24}$$

where m_t and I are to be evaluated locally from their definitions given by Eqs. (20) and (13), respectively.

Solution Procedures

Integral and Exact Solutions

It is desirable to have a simple explicit expression for the functions $-\theta'(0; \lambda)$ so that it can be substituted into Eq. (24) for speedy estimations of heat transfer rates from nonisothermal bodies of arbitrary configuration. Such an expression based on the Karman-Pohlhausen integral method can be derived considering the energy integral equation as follows:

$$\frac{\mathrm{d}}{\mathrm{d}x} r^* \left(\frac{\rho K g_x \beta \Delta T_w}{\mu}\right)^{1/n} \int_0^\delta (T - T_e)^{(n+1)/n}\,\mathrm{d}y = -r^*\alpha \left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{25}$$

We shall also use the following auxiliary relation at the wall implicit in the energy Eq. (6):

$$\left(\frac{\rho K g_x \Delta T_w}{\mu}\right)^{1/n} \frac{\mathrm{d}T_w}{\mathrm{d}x} = \alpha \left.\frac{\partial^2 T}{\partial y^2}\right|_{y=0} \tag{26}$$

Let us assume θ to follow the following function with the parameter ζ:

$$\theta = [1 - (y/\delta)]^\zeta \tag{27}$$

By substituting the foregoing temperature profiles into Eqs. (25) and (26), we obtain two distinct expressions for δ, namely

$$\left(\frac{\delta}{x}\right)^2 Ra_x = 2\zeta\left(1 + \frac{1+n}{n}\zeta\right) I = \frac{\zeta(\zeta - 1)}{m} \tag{28}$$

These two expressions can be combined to determine the unknown parameter ζ as

$$\zeta = \frac{n(1 + 2m_t I)}{n - 2(1+n)m_t I} \tag{29}$$

Knowing ζ, from a given surface temperature, we may evaluate Nu_x from

$$Nu_x = \left(\frac{x}{\delta}\right)\zeta = \sqrt{\frac{n(1 + 2m_t I)}{2(1+2n)I}}\, Ra_x^{1/2} \tag{30}$$

In finding the exact solutions, the Runge-Kutta-Gill integration scheme, together with the Nachtsheim-Swigert iteration scheme, has been employed to integrate Eqs. (16) and (17) satisfying the boundary conditions, Eq. (18), numerically. The upper bound of the integration was considered to be $10 \le \eta \le 12$ for $n = 1$, 1.5, and 2. On the other hand, for $n = 0.5$, this value was taken to be 8 only. In all the above integrations, the acceptable grid size was $\Delta\eta = 0.1$, which was sufficient to get the convergent solutions of the above equations for all the cases treated here.

Throughout, substitution of $n = 1$, the problem reduces to that of Nakayama and Hossain,[15] for the flow of the Newtonian fluid, along a flat plate embedded in a fluid-saturated medium.

Numerical Method

In order to reduce the partial differential Eq. (17) into an ordinary differential equation, we adopt the finite difference scheme described by Keller[13] for ξ or η dependence. To this end we introduce a set of points

$$\xi_0 = 0 \quad \text{and} \quad \xi_i = \xi_{i-1} + k_i \qquad i = 1, 2, \cdots$$

and replace the partial derivatives of f and θ, with respect to ξ and η, by a backward finite difference formula to obtain, for $n = 0.5$

$$f''_{i-1/2} = 2(\theta\theta')_{i-1/2} \tag{31}$$

$$\theta''_{i-1/2} + p_1(\xi_i)(f\theta')_{i-1/2} - p_3(\xi_i)(\theta f')_{i-1/2} = p_3(\xi_i)\alpha_{i-1/2}[f'_{i-1/2}(\theta_i - \theta_{i-1}) - \theta'_{i-1/2}(f_i - f_{i-1})] \tag{32}$$

where $p_1(\xi_i) = \frac{1}{2} - m_t I$, $p_2(\xi_i) = m_t I$, $p_3(\xi_i) = I$, $\alpha_{i-1/2} = \xi_{i-1/2}/k_i$, and the subscript $i - \frac{1}{2}$ refers to

$$(\;)_{i-1/2} = \tfrac{1}{2}[(\;)_i - (\;)_{i-1}]$$

The corresponding boundary conditions become

$$\begin{aligned} \eta = 0: &\quad f_i = 0, \quad \theta_i = 1 \\ \eta \to \infty: &\quad f'_i = \theta_i = 0 \end{aligned} \tag{33}$$

It should be noted that Eqs. (31) and (32) reduce to an algebraic system by a collocation procedure. Specifically, cubic Hermite polynomials are used as interpolations functions for higher order accuracy, which are forced to satisfy the differential equation at Gaussian quadrature points[16]

$$\eta_{j-1} + \left(\frac{1}{2} - \frac{1}{\sqrt{12}}\right) h_j, \qquad \eta_{j-1} + \left(\frac{1}{2} + \frac{1}{\sqrt{12}}\right) h_j$$
$$j = 1, 2, \cdots, J$$

where

$$\eta_0 = 0, \ \eta_j = \eta_j - 1 + h_j \qquad \text{with} \qquad \eta_j = \eta_\infty$$

The graded η spacing is given by

$$h_j = Ch_{j-1} \qquad \text{or} \qquad \eta_j = h_1 \frac{C^j - 1}{C - 1}$$

Given the initial step size h_i, and the ratio of two consecutive step sizes C, the total number of steps can be determined if η_∞ is specified.

Once, solutions for θ and f are known on ξ_{i-1}, the values of $\theta(\xi_i, \eta)$ and $f(\xi_i, \eta)$ are determined by the algebraic system so obtained. This nonlinear system is solved recursively, starting with $i = 0$, employing Newton's quasilinearization technique together with the Keller box method of Keller.[13] The converged solutions on the preceding value of ξ are used as the initial guesses on ξ_i. The convergence criterion was set on $|\delta\theta'| \leq 10^{-6}$. No more than five iterations were required to achieve convergence.

In Eqs. (29–31), with $\alpha_i = 0$, the problem reduces to the case of locally similarity problem for the case when $n = 0.5$. Following the same method, solutions of the problem for other values of n, such as, $n = 1$, 1.5, and 2.0, are obtained with prescribed surface temperature.

In the following section we discuss the results obtained from the above analysis for different cases.

Results and Discussion

Both Eqs. (24) and (30) from the local similarity solution and approximate integral solution hold for all plane and axisymmetric bodies of arbitrary shape. For illustration, similar and nonsimilar problems associated with free convection over a vertical flat plate is considered, assuming various wall temperature distributions in which numerical integration results from some other solution schemes are available.

Exponential Wall Temperature Distribution

In this nonsimilar example, the wall temperature is given by

$$\Delta T_w \propto \exp(\lambda\xi) \tag{34}$$

where $\xi(=x/L)$, L defines characteristic length. Therefore

$$m_t = \lambda\xi \tag{35}$$

$$I = \frac{1 - \exp(-\gamma\lambda\xi)}{\gamma\lambda\xi} \tag{36}$$

$$\gamma = \frac{2n + 1}{n} \tag{37}$$

Equations (35) and (36) are substituted into Eqs. (16) and (17) or (22). The numerical values of the rate of heat transfer in the form of Nu_x are entered into Tables 1–4, respectively, for $n = 0.5$, 1.0, 1.5, 2.0; while $\lambda = 0.5$, 1.0, and 2.0 are obtained by all the methods discussed in the preceeding section for a wide range of ξ in the interval $0.3 \leq \xi \leq 10.0$. From these tables it can be seen that the difference between the numerical values of the Nusselt numbers, for any value of the streamwise distance, obtained from the approximate integration and the finite difference solutions of the locally similarity, as well as the locally nonsimilarity equations, is less than 0.01%.

Table 1 Values of $-\theta'(\xi, 0)/Ra_x^{1/2}$ for $\Delta T_w = \exp(\lambda\xi)$ for $n = 0.5$

ξ	Exact	Approximate	FDLS,[a] Eq. (22)	FDLNS,[b] Eq. (17)
		$\lambda = 0.5$		
0.3	0.460183	0.451362	0.46038	0.46658
0.5	0.513974	0.510144	0.51420	0.52256
0.7	0.566300	0.565490	0.59213	0.60237
1.0	0.641690	0.643529	0.71360	0.72547
3.0	1.054654	1.061538	1.05517	1.06778
5.0	1.360376	1.369327	1.36104	1.37207
10.0	1.923829	1.936492	1.92476	1.93334
		$\lambda = 1.0$		
0.3	0.540334	0.538193	0.54058	0.54984
0.5	0.641690	0.643529	0.64200	0.65320
0.7	0.736279	0.740042	0.78147	0.79394
1.0	0.866050	0.871395	0.98337	0.99612
3.0	1.490196	1.500003	1.49092	1.50129
5.0	1.923829	1.936492	1.92476	1.93334
10.0	2.720705	2.738613	2.72203	2.72865
		$\lambda = 2.0$		
0.3	0.689826	0.692770	0.69016	0.70234
0.5	0.866050	0.871395	0.86647	0.87966
0.7	1.019335	1.025962	1.08945	1.10220
1.0	1.216882	1.224882	1.38798	1.39911
3.0	2.107451	2.121320	2.10847	2.11666
5.0	2.720707	2.738613	2.72203	2.72879
10.0	3.847661	3.872983	3.84953	3.85478

[a]Finite difference solutions for locally similar equation.
[b]Finite difference solutions for locally nonsimilar equation.

Table 2 Values of $-\theta'(\xi, 0)/Ra_x^{1/2}$ for $\Delta T_w = \exp(\lambda\xi)$ for $n = 1.0$

ξ	Exact	Approximate	FDLS, Eq. (22)	FDLNS, Eq. (17)
		$\lambda = 0.5$		
0.3	0.522628	0.506922	0.52246	0.52967
0.5	0.574459	0.565898	0.57423	0.58413
0.8	0.650213	0.647716	0.64990	0.66218
1.0	0.699090	0.698907	0.69873	0.71206
3.0	1.115983	1.121796	1.11534	1.13312
5.0	1.435961	1.443615	1.43514	1.45310
10.0	2.030404	2.041242	2.02924	2.04514
		$\lambda = 1.0$		
0.3	0.600010	0.593942	0.59975	0.61073
0.5	0.699090	0.698907	0.69873	0.71218
0.8	0.837359	0.840580	0.83689	0.85226
1.0	0.922821	0.927109	0.92230	0.93859
3.0	1.572806	1.581197	1.57190	1.58953
5.0	2.030404	2.041242	2.02924	2.04513
10.0	2.871428	2.886751	2.86978	2.88322
		$\lambda = 2.0$		
0.3	0.746574	0.747937	0.74617	0.76072
0.5	0.922821	0.927109	0.92230	0.93894
0.8	1.151534	1.157572	1.15087	1.16897
1.0	1.285133	1.291956	1.28439	1.30272
3.0	2.224199	2.236068	2.22292	2.23831
5.0	2.871428	2.886751	2.86978	2.88333
10.0	4.060812	4.082483	4.05848	4.06989

Table 3 Values of $-\theta'(\xi, 0)/Ra_x^{1/2}$ for $\Delta T_w = \exp(\lambda\xi)$ for $n = 1.5$

ξ	Exact	Approximate	FDLS, Eq. (22)	FDLNS, Eq. (17)
		$\lambda = 0.5$		
0.3	0.552879	0.532676	0.55299	0.56020
0.5	0.604088	0.592152	0.60427	0.61426
0.8	0.679215	0.674499	0.67952	0.69185
1.0	0.727861	0.725939	0.72825	0.74152
3.0	1.147174	1.151735	1.14799	1.16531
5.0	1.473511	1.479558	1.47456	1.49304
10.0	2.083086	2.091651	2.08458	2.10184
		$\lambda = 1.0$		
0.3	0.629390	0.620399	0.62961	0.64055
0.5	0.727861	0.725939	0.72825	0.74161
0.8	0.866111	0.868183	0.86669	0.88171
1.0	0.951975	0.955163	0.95263	0.96854
3.0	1.613709	1.620341	1.61486	1.63348
5.0	2.083086	2.091651	2.08458	2.10189
10.0	2.945926	2.958040	2.94804	2.96299
		$\lambda = 2.0$		
0.3	0.775239	0.775172	0.77570	0.79003
0.5	0.951975	0.955163	0.95263	0.96886
0.8	1.183269	1.188016	1.18411	1.20214
1.0	1.319324	1.324708	1.32027	1.33886
3.0	2.281905	2.291288	2.28354	2.30038
5.0	2.945927	2.958040	2.94804	2.96309
10.0	4.166170	4.183300	4.16916	4.18200

Power Law Wall Temperature Distribution

In this example, the wall temperature distribution is assumed as follows:

$$\Delta T \propto 1 + \xi^{\lambda} \tag{38}$$

Hence

$$m_t = \frac{\lambda \xi^{\lambda}}{1 + \xi^{\lambda}} \tag{39}$$

Table 4 Values of $-\theta'(\xi, 0)/Ra_x^{1/2}$ for $\Delta T_w = \exp(\lambda\xi)$ for $n = 2.0$

ξ	Exact	Approximate	FDLS, Eq. (22)	FDLNS, Eq. (17)
		$\lambda = 0.5$		
0.3	0.571096	0.547575	0.57086	0.57817
0.5	0.622228	0.607428	0.62189	0.63193
0.8	0.697416	0.690214	0.69692	0.70925
1.0	0.746181	0.741881	0.74559	0.75884
3.0	1.168117	1.169642	1.16713	1.18446
4.0	1.342274	1.344167	1.34115	1.35963
6.0	1.641058	1.643420	1.63969	1.65874
10.0	2.118268	2.121323	2.11651	2.13461
		$\lambda = 1.0$		
0.3	0.647531	0.635837	0.64713	0.65824
0.5	0.746181	0.741881	0.74559	0.75894
0.8	0.884987	0.884651	0.88423	0.89902
1.0	0.971337	0.971963	0.97050	0.98609
3.0	1.641058	1.643420	1.63969	1.65883
5.0	2.118268	2.121323	2.11651	2.13462
10.0	2.995680	3.000000	2.99319	3.00911
		$\lambda = 2.0$		
0.3	0.793717	0.791306	0.79306	0.80731
0.5	0.971337	0.971963	0.97050	0.98640
0.8	1.204587	1.206203	1.20357	1.22138
1.0	1.342274	1.344167	1.34115	1.35978
3.0	2.320444	2.323790	2.31851	2.33619
5.0	2.995680	3.000000	2.99319	3.00919
10.0	4.236532	4.242641	4.23301	4.24689

$$I(\xi) = \frac{\int_0^{\xi} (1 + z^{\lambda})^{\gamma}\, dz}{\xi(1 + \xi^{\lambda})^{\gamma}} \tag{40}$$

As before, relations (39) and (40) are substituted into the expressions (16) and (22). The results in terms of $Nu_x/Ra_x^{1/2}$ are plotted in Fig. 2, for different values of $\lambda = 0.5, 1.0, 2$, and $n = 0.5, 1.0, 2.0$, against ξ ranging between 0.0–10.0. In this figure, all the curves for $n = 1.0$ represent the local rate of heat transfer in Newtonian fluid, which has already been discussed by Nakayama and Hossain.[15] In this figure, the circled curve, which represents the solutions obtained by employing the finite difference method, show a good agreement with the solutions obtained by other methods. From the other curves for $n = 0.5$ and 2.0, it can be seen that the performance of the approximate integral as well as the finite difference methods appear satisfactory with respect to the exact solutions. Due to the flow acceleration caused by the streamwise increase of wall temperature, the heat transfer increases near the leading edge and then approaches the asymptotic values with increasing streamwise distance. The asymptotic value increases with the increase of λ at every value of the parameter n.

Sinusoidal Wall Temperature Distribution

In this last example, the wall temperature distribution is taken as

$$\Delta T \propto \sin \xi \tag{41}$$

such that

$$m_t = \frac{\xi}{\tan \xi} \tag{42}$$

$$I(\xi) = \frac{\int_0^{\xi} \sin z\, dz}{\xi \sin \xi} \tag{43}$$

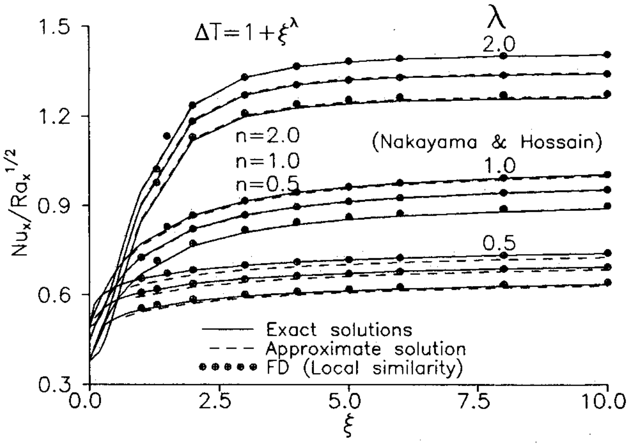

Fig. 2 Heat transfer results for power-law variation in wall temperature for n = 0.5, 1.0, and 2.0.

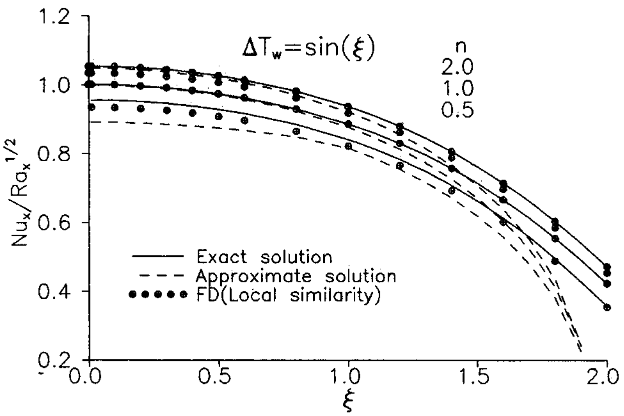

Fig. 3 Heat transfer results for sinusoidal variation in wall temperature for n = 0.5, 1.0, and 2.0.

In Fig. 3, the present integral approximate and the finite difference solutions are compared with those of the exact solutions of the locally similar equation of the energy distribution. From this figure it can be seen that the rate of heat transfer function $Nu_x/Ra_x^{1/2}$ decreases with the increasing streamwise distance. We further observe that for all the values of n, the finite difference solutions agree completely with the exact solutions in the whole region of the parameter ξ. It can further be observed that the results from the approximate integration are close to the other solutions near the leading edge, and the difference that occurs in the downstream is appreciable, since in this region the flow deceleration takes place.

Conclusions

Two distinct solution methods, e.g., the shooting and the two-point finite difference methods on the local similarity equations and an integral method, have been employed on the nonsimilar free convection thermal boundary-layer flow over a body of arbitrary shape embedded in a non-Newtonian fluid-saturated porous medium. For illustration, numerical results with and without local similarity assumptions are presented for the case of vertical plate with exponential, power-law, and sinusoidal surface temperature distributions. From the present analysis it has been confirmed that both finite difference and the integral approximate solutions perform well with the exact solutions for all wall temperature distributions considered here. In fact, the only difference between the results form the integral solutions with the other two occurs in the downstream region of the plate subject to the sinusoidal wall temperature distribution.

References

[1]Cheng, P., and Minkowycz, W. J., "Free Convection About a Vertical Plate in a Saturated Porous Medium with Application to Heat Transfer from a Dike," *Journal of Geophysical Research*, Vol. 82, 1977, pp. 2040–2044.

[2]Cheng, P., "Heat Transfer in Geothermal Systems," *Advances in Heat Transfer*, Vol. 14, 1978, pp. 1–105.

[3]Nield, D. A., and Bejan, A., *Convection in Porous Media*, Springer-Verlag, New York, 1992.

[4]Chen, H. T., and Chen, C. K., "Free Convection of Non-Newtonian Fluids Along a Vertical Plate Embedded in a Porous Medium," *Journal of Heat Transfer*, Vol. 110, 1988a, pp. 257–260.

[5]Chen, H. T., and Chen, C. K., "Natural Convection in Non-Newtonian Fluid About a Horizontal Cylinder and a Sphere in a Porous Medium," *International Communication of Heat and Mass Transfer*, Vol. 15, 1988b, pp. 605–615.

[6]Christopher, R. V., and Middleman, S., "Power-Law Flow Through a Packed Tube," *Industrial and Engineering Chemistry Fundamentals*, Vol. 4, No. 4, 1965, pp. 422–426.

[7]Nakayama, A., and Koyama, H., "Buoyancy Induced Flow of Non-Newtonian Fluids over a Non-Isothermal Body of Arbitrary Shape in a Fluid-Saturated Porous Medium," *Applied Scientific Research*, Vol. 48, 1991, pp. 55–70.

[8]Nakayama, A., and Shenoy, A. V., "A Unified Similarity Transformation for Darcy and Non-Darcy Forced, Free and Mixed Convection Heat Transfer in Non-Newtonian Inelastic Fluid Saturated Porous Media," *The Chem. Engng. J.*, Vol. 50, 1992, pp. 33–45.

[9]Sparrow, E. M., and Yu, H. S., "Local Non-Similarity Thermal Boundary Layer Solutions," *Transactions of the American Society of Mechanical Engineers Journal of Heat Transfer*, Vol. 93, 1971, pp. 328–334.

[10]Dharmadhikari, R. V., and Kale, D. D., "Flow of Non-Newtonian Fluid Through Porous Media," *Chem. Engng. Scie.*, Vol. 40, 1985, pp. 527–529.

[11]Wang, C., and Tu, G., "Boundary Layer Flow and Heat Transfer on Non-Newtonian Fluids in Porous Media," *Int. J. Heat Fluid Flow*, Vol. 10, 1989, pp. 160–165.

[12]Nachtsheim, P. R., and Swigert, P., "Satisfaction of Asymptotic Boundary Conditions in Numerical Solution of Systems of Non-Linear Equation of Boundary Layer Type," NASA TN D-3004, Oct. 1965.

[13]Keller, H. B., "Numerical Methods in Boundary Layer Theory," *Annual Review of Fluid Mechanics*, Vol. 10, 1978, pp. 417–433.

[14]Nakayama, A., and Koyama, H., "Free Convection Heat Transfer over a Non-Isothermal Body of Arbitrary Shape Embedded in a Fluid Saturated Porous Medium," *Journal of Heat Transfer*, Vol. 109, 1987, pp. 125–130.

[15]Nakayama, A., and Hossain, M. A., "Free Convection in a Saturated Porous Medium Beyond the Similarity Solution," *Applied Scientific Research* (to be published).

[16]Yücel, A., "The Influence of Injection or Withdrawal of Fluid on Free Convection About a Vertical Cylinder in a Porous Medium," *Numerical Heat Transfer*, Vol. 7, 1984, pp. 483–493.